Note* 1555

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A theory of hubs, ruins, and blockers

1 Basic Notions

We assume the universe has countably many individuals, $i_0, i_1, \ldots \in \mathbb{I}$. We make time discrete, so $t \in \mathbb{N}$, however we remain agnostic about temporal resolution.[1] As the theory develops and inconsistencies are discovered, the temporal aspect will be further developed.

The meet relation ($\bowtie$) represents meetings between individuals at a particular time, so A $\bowtie_t B$ means that A met B at time t. Note that we deliberately leave ambiguous how long a meeting must last for it to be considered a meeting. We require that $\bowtie$ commutes:

$$A \bowtie_t B \;\equiv\; B \bowtie_t A \tag{1}$$

This makes intuitive sense since for a meeting to have occurred both parties must have considered it a meeting. We further require that individuals engaging in a meeting must be spatially close to each other, assuming individuals are points inhabiting $\mathbb{R}^2$. $\mathcal{L}_t(A) \in \mathbb{R}^2$ gives the cartesian coordinates of individual A at time t. This then gives:

$$A \bowtie_t B \;\Rightarrow\; \mathcal{L}_t(A) \approx \mathcal{L}_t(B) \tag{2}$$

Where

$$(x, y) \approx (x', y') \;\equiv\; \sqrt{(x - x')^2 + (y - y')^2} \le \epsilon \tag{3}$$

for some sensible ϵ which ensures individuals are within speaking (and handshaking) distance but not literally inside each other. Note that the converse for equation (2) need not necessarily hold. For instance, two individuals can be arbitrarily close for a long period of time but still not have met. (See especially Definition 1.3.)

Intuitively, a *hub* is an individual who enables meetings between individuals who have not previously met.

Definition 1.1. *$A \bowtie_t B$ is called a* ***fresh meet****, written $A \bowtie_t^* B$, if there does not exist $t' < t$ such that $A \bowtie_{t'} B$.*

Definition 1.2 (Hub)**.** *Suppose for all $t < t'$, $A \not\bowtie_t B$, but at t' the following conditions hold:*

1. $A \bowtie_{t'} C$
2. $B \bowtie_{t'} C$
3. $A \bowtie_{t'}^* B$

Then C is said to have ***acted as a hub*** *for A and B at t.*

A *ruin*, however, is an individual who, given a situation where a meet is possible between individuals who haven't met, doesn't enable the meet.

Definition 1.3 (Ruin)**.** *Suppose for all $t < t'$, $A \not\bowtie_t B$, but at t' the following conditions hold:*

1. $A \bowtie_{t'} C$
2. $B \bowtie_{t'} C$
3. $A \not\bowtie_{t'} B$

Then C is said to have ***acted as a ruin*** *for A and B at t.*

There exists another class of individuals which poses similar difficulties as a ruin for a successful meeting to take place. A *blocker* of A and B tries to maximise the distance between A and B.

Definition 1.4 (Blocker)**.** *Suppose for all $t < t'$, $A \not\bowtie_t B$, but for some t_A, $A \bowtie_{t_A}^* C$ and for some t_B, $B \bowtie_{t_B}^* C$. If $\mathcal{L}_t(B) \approx \mathcal{L}_t(C)$ but $\mathcal{L}_t(A) \not\approx \mathcal{L}_t(B)$, then C has* ***blocked*** *A from being proximal to B at t.*

*Notes in this series are for ϵ-baked ideas, for $1 \ge \epsilon \ge 0$. Only exceptionally should they be cited or distributed outwith the Mathematical Reasoning Group.

[1] $<$, $>$, and $=$ for $\mathbb{N}$ give properties equivalent to using P and O in Walker's axioms. There are objections to the psychological validity of such axioms.

Theorem 1.1. *All blockers are ruins.*

Proof. If C blocks A from being proximal to B at time t, then $\mathcal{L}_t(A) \not\approx \mathcal{L}_t(B)$. Thus by (2), $A \not\bowtie_t B$. □

Theorem 1.2. *Not all ruins are blockers.*

Proof. Trivial, since Definition 1.3 remains silent on proximity, it's possible for $\mathcal{L}_t(A) \approx \mathcal{L}_t(B)$ but still all the conditions for ruin to hold. □

Theorem 1.3. *A hub A can act as a hub for itself with B iff A has not previously met B.*

Proof. Let A and B be two individuals and suppose A has acted as a hub for A and B. So for all $t < t'$, $A \not\bowtie_t B$. At t' the following conditions hold: $A \bowtie_{t'} A$, $A \bowtie_{t'} B$, and $A \bowtie^*_{t'} B$. For the last two conditions to hold, A must not previously have met B (by Definition 1.1). □

2 Desire

So far we have considered all classes of meets, excluding the intentions of individuals. To begin the modelling of this, we must introduce a *desires-to-meet* relation, $\heartsuit$.

Definition 2.1. *If $A \heartsuit_t B$ but $B \not\heartsuit_t A$, then A suffers from **asymmetrical desire** of B.*

Definition 2.2. *If $A \heartsuit_t B$ and $B \heartsuit_t A$, then A **symmetrically desires** B, written $A \heartsuit^+_t B$.*

Desire is non-monotonic.

$$t_1 > t_2 \ \wedge \ A \heartsuit_{t_1} B \ \not\Rightarrow \ A \heartsuit_{t_2} B \tag{4}$$

Trivially, symmetrical desire commutes:

$$A \heartsuit^+_t B \ \equiv \ B \heartsuit^+_t A \tag{5}$$

This allows us to describe the intentions of individuals and thus now we can solve the problem of specifying for whom a hub has been useful.

Definition 2.3 (Useful hub)**.** *A hub C is said to be a **useful** A-B-**hub for** A **at** t' if the following conditions hold.*

1. *C has acted as a hub for A and B at t, i.e. there exists a t such that $A \bowtie^*_t B$, along with $A \bowtie_t C$ and $B \bowtie_t C$.*
2. *For some $t' > t$, $A \bowtie_{t'} B$, but $A \not\bowtie_{t'} C$ and $B \not\bowtie_{t'} C$.*
3. *$A \heartsuit_{t'} B$.*

More astute readers may have noticed some subtle points about this definition. It is possible for someone to act as a useful hub for A between A and B at a time when $A \not\heartsuit_t B$.[2] C, aware of this, but also aware that A is likely to, given more sentential stimuli, develop desire for B, can still engineer the situation in a manner which allows a subsequent meet.

3 Conclusions

I hope the reader has not failed to grasp the extreme importance of this functional description of hubs, ruins, and blockers. Future work includes a model theoretic interpretation, incorporation of a formalisation of psychometrics tests, and the implementation of a portable proof procedure.

[2] For instance B could have The First Sentence Problem, a common social affliction.