

How people converge on an interpretation of conditionals¹

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Plan for today

- ① How do people interpret conditionals?
- ② Probabilistic truth table tasks: the theory and data so far
- ③ How do working memory processes relate to conditional interpretation?

Probabilistic truth table tasks

(Evans, Handley, & Over, 2003; Oberauer & Wilhelm, 2003)

- Cards drawn randomly from a shuffled deck
- Each card has an element on two dimensions
- Joint frequencies are provided numerically, e.g.,

1 yellow circle

4 yellow diamonds

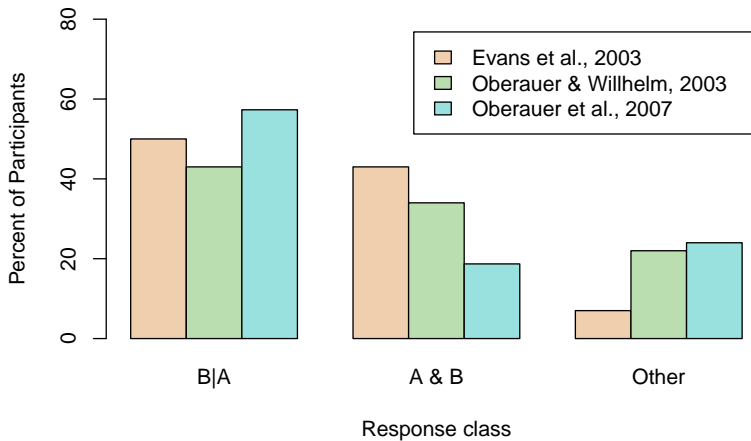
16 red circles

16 red diamonds

- Task: what is the probability that

If the card is yellow then it has a circle printed on it?

The typical spread of results



Semantic values

A	B	$A \supset B$	$A \wedge B$	$B A$
true	true	true	true	true
true	false	false	false	false
false	true	true	false	void
false	false	true	false	void

A	B	$A \supset B$	$A \wedge B$	$B A$
true	true	true	true	true
true	false	false	false	false
false	true	true	false	void
false	false	true	false	void

- In probability logic, $B|A$ is a fully respectable connective
- Not only defined for conditional probability $P(B|A)$
- Same semantic values as the 'defective' conditional
- Justifiably not defective!

Conditional event: a standardized conjunction

- Based on the Ramsey (1929/1990) test
- Hypothetically assume A , fix degree of belief in B
- Evans et al. (2003): compare $P(A \wedge B)$ with $P(A \wedge \neg B)$

Explanations for responses

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- Evans et al. (2003): incomplete Ramsey test

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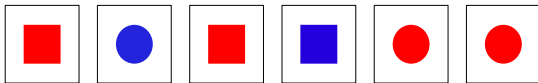
Conjunction: a 'permissible' interpretation

Edgington (2003):

- 1 Consider when 'if A , then B ' gets truth value *true*
- 2 What's the probability of this? $P(A \wedge B)$

Our task

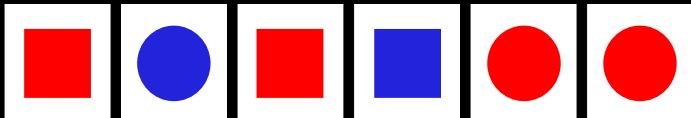
(Fugard, Pfeifer, Mayerhofer, & Kleiter, 2009)



- Graphical, not numerical
- Hoped that all frequencies, joint (e.g., *square and red*) and marginal (e.g., *red*), equally obtainable by counting
- Systematic enumeration of all 84 joint frequencies:
($A \wedge B$, $A \wedge \neg B$, $\neg A \wedge B$, $\neg A \wedge \neg B$)
- So more items than typically used
- Responses of form '*x out of y*' rather than probabilities, again minimizing arithmetic (and making scoring easier for us)

Hier ist der Würfel 1

Seine Seiten sehen so aus:



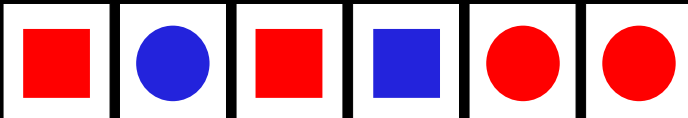
Der Würfel wird geworfen. Eine zufällige Seite zeigt nach oben.
Wie sicher können Sie sein, dass der folgende Satz stimmt?

Wenn die Seite ein Viereck zeigt, dann zeigt die Seite rot.

aus

Here is die 1

Its sides look like this:



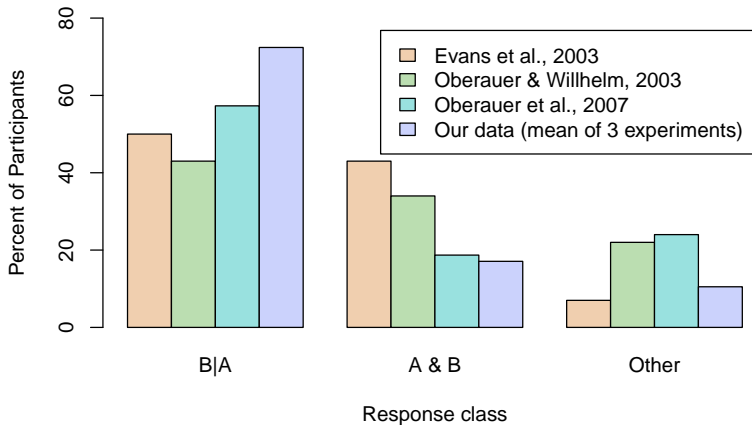
The die is thrown. A random side shows up.
How sure can you be that the following statement holds?

If the side shows a square, then the side shows red.

out of

How did our results compare?

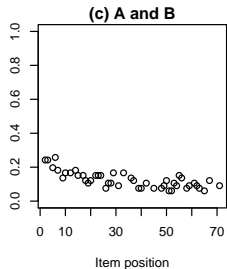
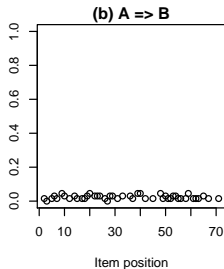
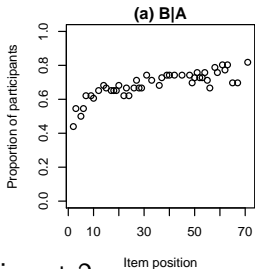
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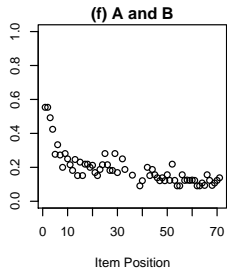
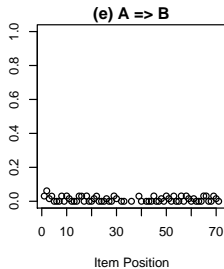
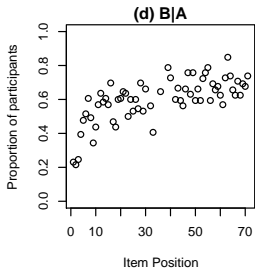
Converging

(Fugard, Pfeifer, Mayerhofer, & Kleiter, 2009)

Experiment 1



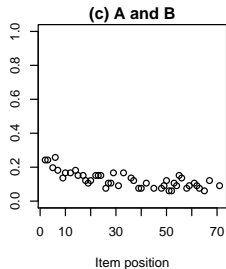
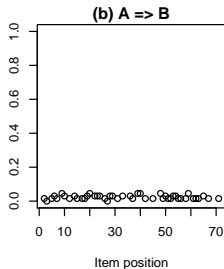
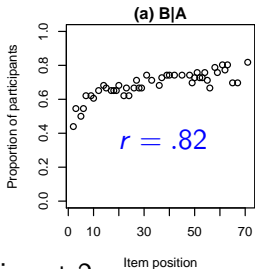
Experiment 2



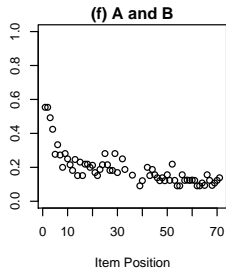
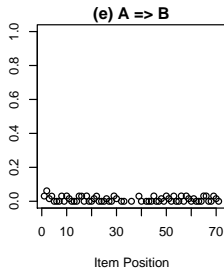
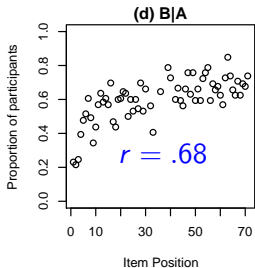
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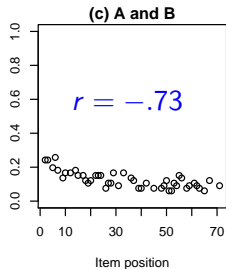
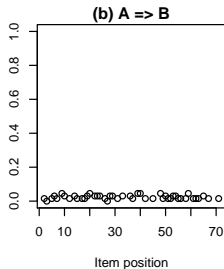
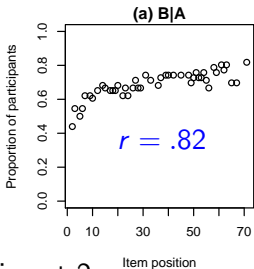
Experiment 2



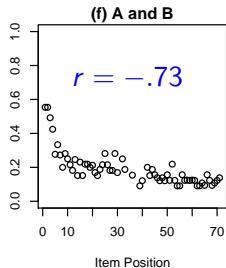
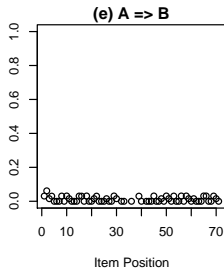
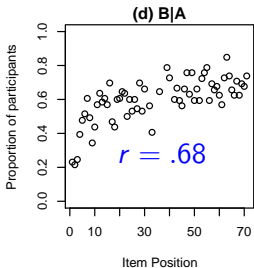
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Experiment 1



Experiment 2



Why?

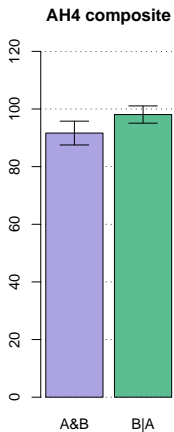
- Insight effect (even with some 'aha!' reports)
- Signs that the goal of the cognition changed, not that reasoning became easier
- Shift from heuristic to analytic processing after task adaptation, e.g., reduced WM load
- Time to disentangle dimensions?

Why?

- Insight effect (even with some 'aha!' reports)
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- Shift from heuristic to analytic processing after task adaptation, e.g., reduced WM load
- Time to disentangle dimensions?
- Some elaboration would be lovely. . .

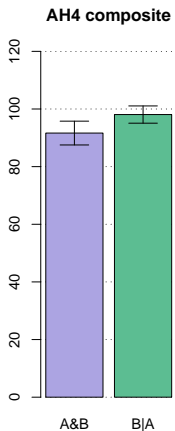
First clue: an IQ composite is a predictor of interpretation

(Evans, Handley, Neilens, & Over, 2007)



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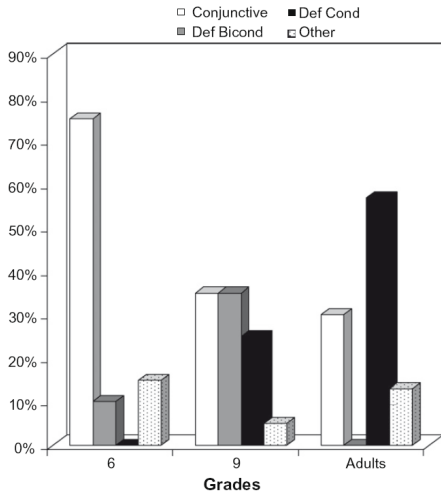
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- Used a composite from the AH4 test:
 - **Verbal-analytic** reasoning, e.g., analogies, number series
 - **Visuospatial**, e.g., reasoning about shapes
- Interpreted as reflecting **working memory capacity**
- Mental models theory predicts those with higher WM go for material conditional
- Even Byrne and Johnson-Laird (2009, p. 286) note that this result 'is a conundrum' for their mental model theory.
- Lower ability: 'shallower processing'

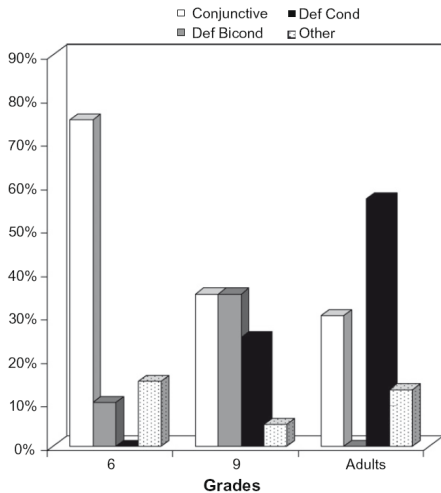
Second clue: developmental shift from $A \wedge B$ to $B|A$

(Gauffroy & Barrouillet, 2009)



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- Working memory capacity explanation back again
- New version of mental models theory:

dual-processes
+
explicit representation of
void

The link with WM?

Barrouillet et al. (2008), with notation by Oberauer and Oaksford (2008)

Heuristic process



A B



A B



A B

The link with WM?

Barrouillet et al. (2008), with notation by Oberauer and Oaksford (2008)

Analytic process



$A \quad B$



$A \quad B$
($\neg A \quad \neg B$)



$A \quad B$
($\neg A \quad \neg B$)
($\neg A \quad B$)

Developmental sequence of 'if A , then B ' interpretation

1. $A \wedge B \mid \Omega$
2. $A \wedge B \mid A \vee B$
3. $A \wedge B \mid A$

Developmental sequence of 'if A , then B ' interpretation

1. $A \wedge B \mid \Omega \quad \equiv \quad A \wedge B$ Conjunction
2. $A \wedge B \mid A \vee B \quad \equiv \quad A \dashv\vdash B$ 'Defective' biconditional
3. $A \wedge B \mid A \quad \equiv \quad B \mid A$ 'Defective' conditional

The present study

- Attempt to replicate **shifts** of interpretation
- Fishing for **biconditional events**
- Directly test the influence of **working memory processes** on interpretation and shifts

Experiment

- Individual testing, paid
- Self-paced
- Computer controlled
- Response times collected
- 55 students (25 females and 30 males)
- Aged 18–32 ($M = 22.7$; $SD = 2.8$)
- No psychologists or people trained in logic

The Card Task

Enumerate items

- Two binary dimensions
- Joints represented as 2×2 table with grand total, n
- There are $(n + 1)(n + 2)(n + 3)/6$ of them
- Enumerate by recursion over flattened table length

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Take subset which distinguishes interpretations

$$B|A \quad A|B$$

$$A \supset B \quad B \supset A$$

$$A \wedge B$$

$$A \dashv\vdash B \quad \equiv \quad B|A \wedge A|B$$

$$A = B \quad \equiv \quad A \supset B \wedge B \supset A$$

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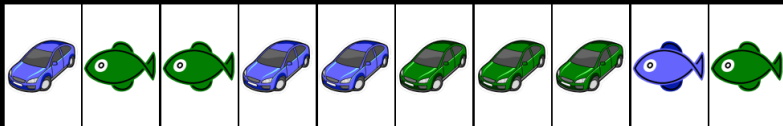
$$A = B \quad \equiv \quad A \supset B \wedge B \supset A$$

Final selection

10 cards, 286 arrangements, 52 items which distinguish

Hier ist der Kartenstapel 9

Die Karten sehen so aus:

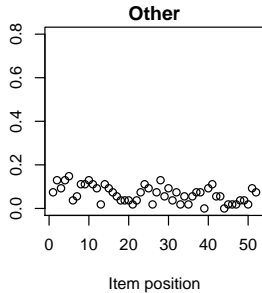
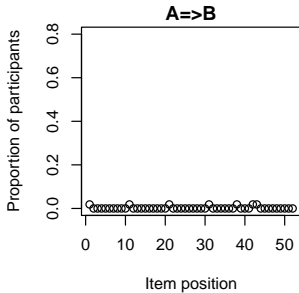
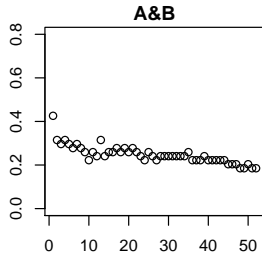
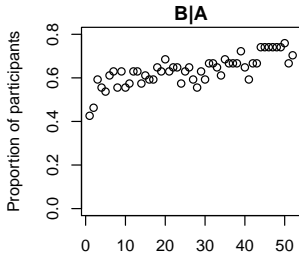


Die Karten werden gemischt. Eine zufällige Karte wird gezogen.
Wie sicher können Sie sein, dass der folgende Satz stimmt?

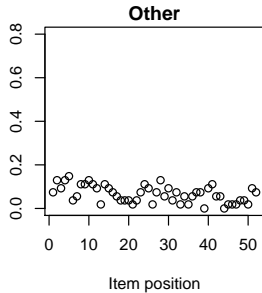
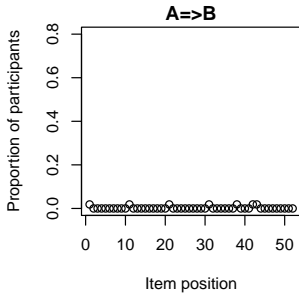
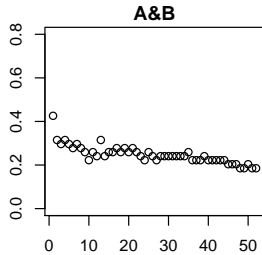
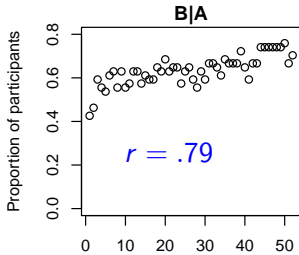
Wenn die Karte ein Auto zeigt, dann zeigt die Karte blau

aus

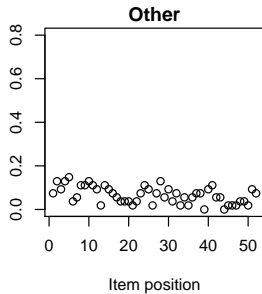
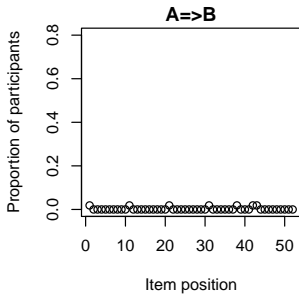
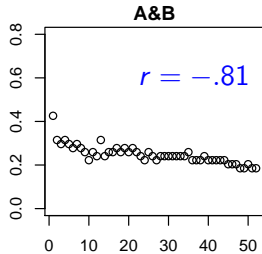
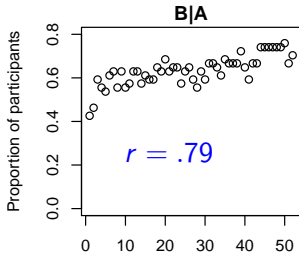
Convergence again



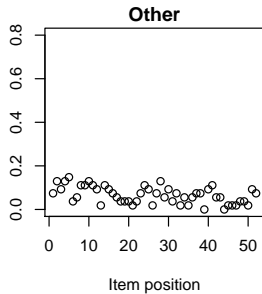
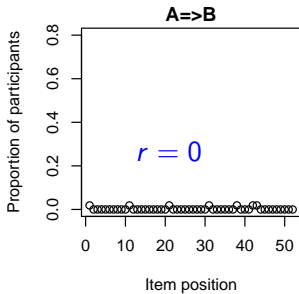
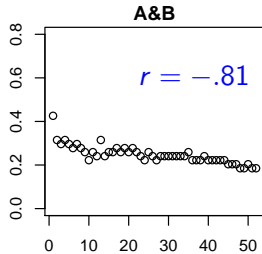
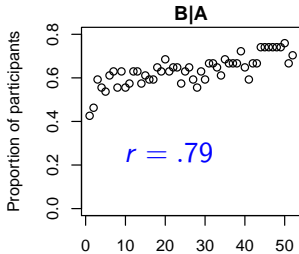
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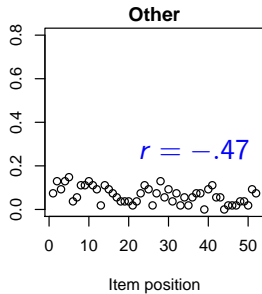
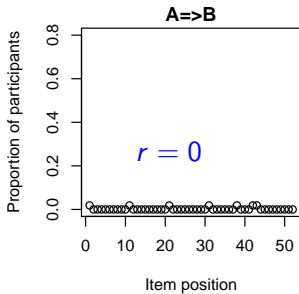
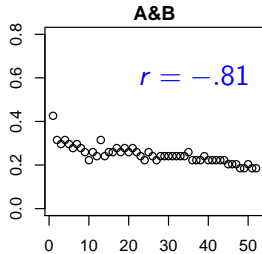
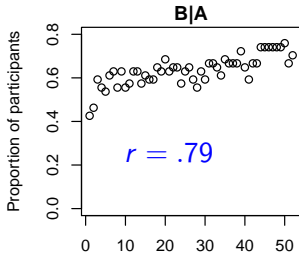
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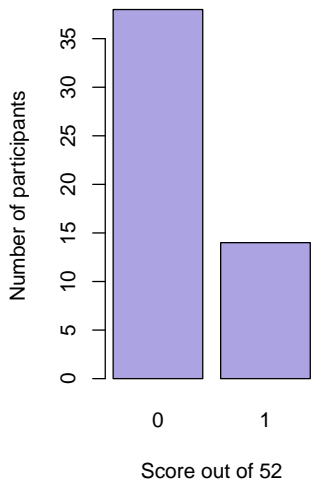
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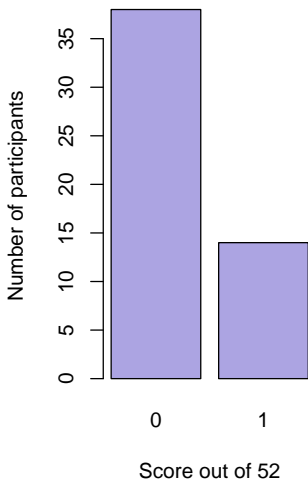
Convergence again



Any biconditional events?



Any biconditional events?



Not many!

Changepoints

- Is there evidence of qualitative shifts of interpretation?
- Multinomial regression for each participant
- Every putative changepoint
- Each model:

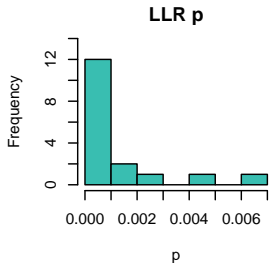
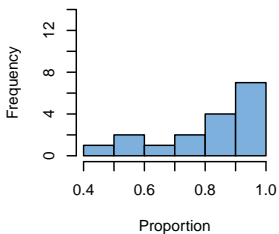
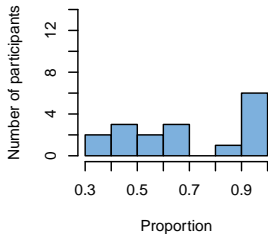
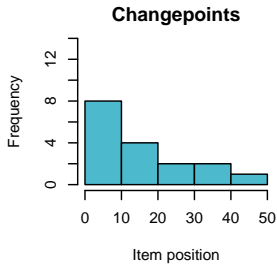
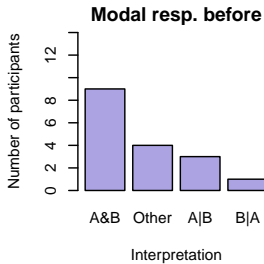
Outcome = Interpretation category

Predictor = $\begin{cases} 1, & \text{if item position} \geq \text{putative changepoint;} \\ 0, & \text{otherwise.} \end{cases}$

- Compare null model with each changepoint model by log-likelihood ratio (LLR)
- Select changepoint by model with highest LLR

Changepoints: summary

Selected if $\Delta AIC < -6$: around 30% of participants

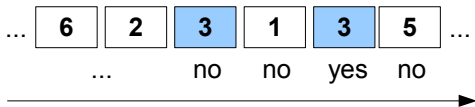


Working memory

- Working memory \neq a passive bucket of data
- Rapid encoding
- Robust maintenance
- Multiple, separate, representations
- Selective updating
- Idea of **proactive** versus **reactive** control
(e.g., Braver, Gray, & Burgess, 2007)
- A range of properties to characterize beyond capacity

2-back task

Adapted from Kane, Conway, Miura, and Colflesh (2007)



- Sequence of numerals shown sequentially one at a time
- Compare current numeral with numeral seen two steps before
- If they are the same, then press 'yes', otherwise 'no'

6

2

3

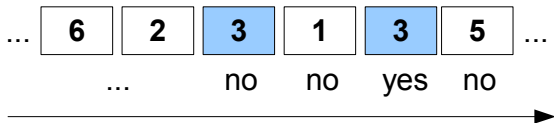
1

3

5

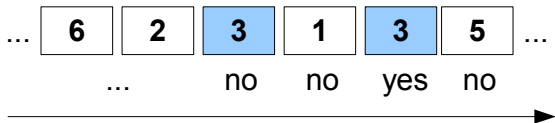
n -back conditions

Target 2-back (18 trials)

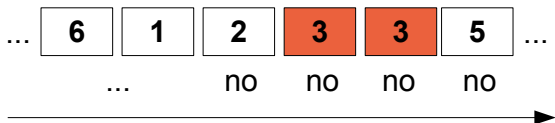


n -back conditions

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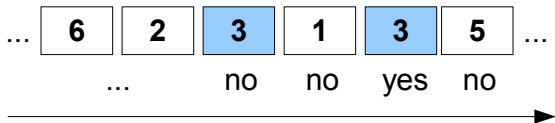


Lure 1-back (9 trials)

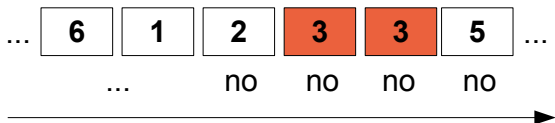


n -back conditions

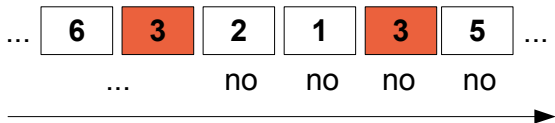
Target 2-back (18 trials)



Lure 1-back (9 trials)

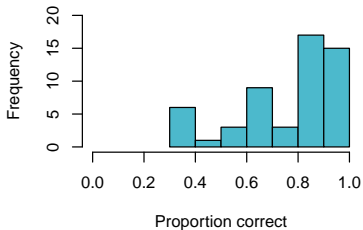


Lure 3-back (9 trials)

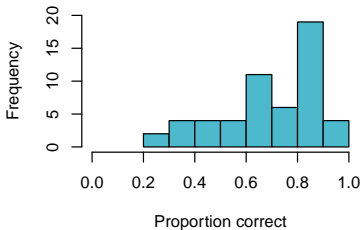


Results: n -back descriptives

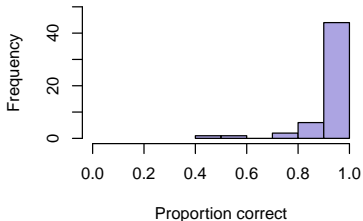
(a) 1-back lure



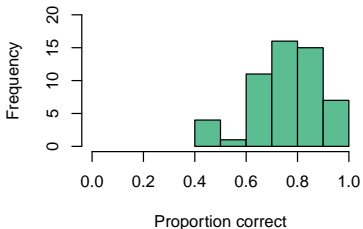
(b) 3-back lure



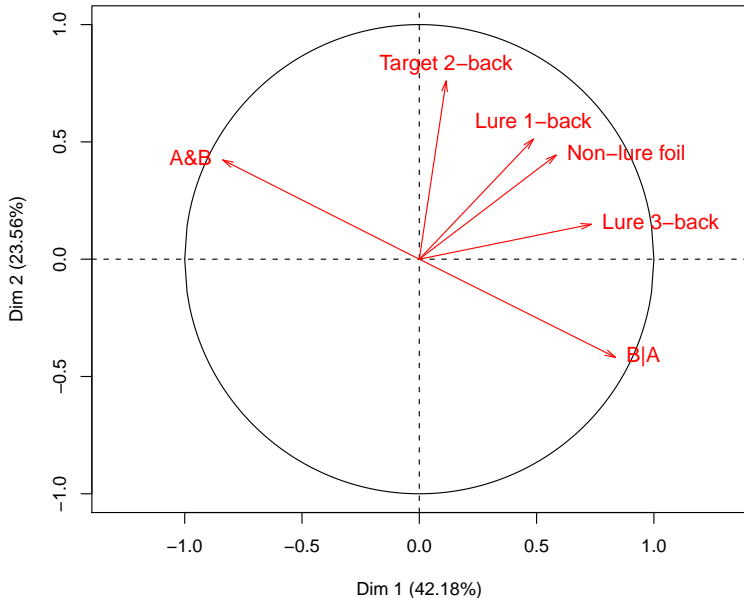
(c) Non-lure foil



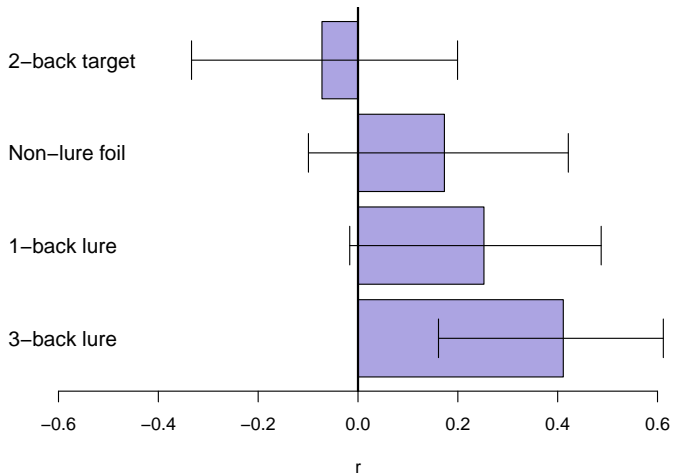
(d) 2-back target



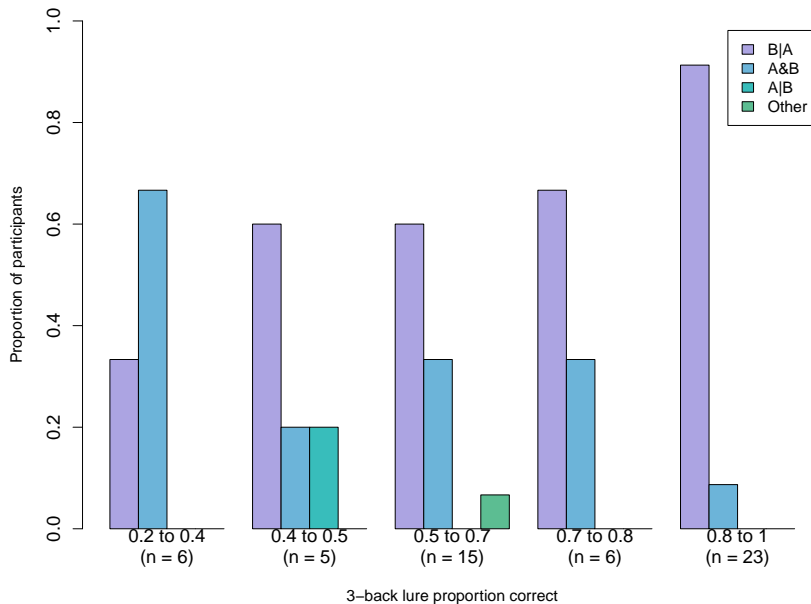
n -back accuracy and interpretation scores



n -back accuracy and conditional event score

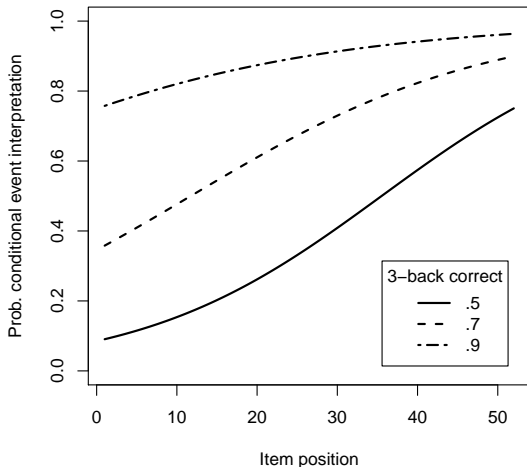


Modal interpretation as a function of 3-back lure



When people converge moderated by lure performance

Logistic mixed effects model



- Item position
 $z = 4.1, p < .001$
- 3-back lure accuracy
 $z = 3.6, p < .001$
- position \times lure
 $z = -2.1, p = .038$

Tip-toeing towards a process model

$A \wedge B$ responses

- Due to more **bottom-up, reactive** control
- Remember antecedent and consequent
- **Cued** by corresponding shapes on the cards

$B|A$ responses

- Due to more **top-down, proactive** control
- **Separate** the **dimensions**
- Store antecedent, consequent, and their sequential order
- **Goal-directed search** for appropriate shapes and colors

A conundrum: causal and counterfactual tasks

Over, Hadjichristidis, Evans, Handley, and Sloman (2007)

- Ask people for joint probabilities, e.g.,
 - $P(\text{global warming continues} \wedge \text{London will be flooded})$
 - $P(\text{global warming continues} \wedge \neg \text{London will be flooded})$
 - $P(\neg \text{global warming continues} \wedge \text{London will be flooded})$
 - $P(\neg \text{global warming continues} \wedge \neg \text{London will be flooded})$
- Then ask for
 $P(\text{if global warming continues, then London will be flooded})$

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The authors note:

*'... the influence of conjunctive probability is a lot weaker for these **realistic** conditionals'*

But see also Evans and Over (2004, p. 80) on the selection task:

*'... all **realistic** versions of the selection task that reliably facilitated performance also **subtly altered the logic of the task**...'*

Why the difference between tasks?

- Here **joint probabilities elicited** rather than presented
- Common cognitive representation generates both joints and $P(B|A)$, rather than $P(B|A)$ being inferred from the joints?
- Reduced bottom-up cueing?

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- Reduced bottom-up cueing?

However. . . Evans et al. (2008)

- Very few conjunctions on a non-probabilistic task with casual conditionals
- 'when conditionals are **realistic**, people do not confuse them with conjunctions'
- But now there are more biconditional events!
- Especially for those with lower AH4 composite score

- Replication of the within-participant shift
- Conditional event responses more likely for those with better top-down WM control
- When people converge also moderated by WM performance
- Still work to do relating indicative with causal and counterfactual tasks!



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- Thanks to Angelo Gilio, Daniela Kloo, David Over, Eva Rafetseder, Guilherme Wood, and Leonhard Kratzer for helpful comments

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